Chain Rule

1. Suppose that f and g are differentiable functions. Also suppose that f is a decreasing function and g is an increasing function. What can you conclude about $\frac{d}{dx} f\left(g(x)\right)$? Explain.

2. If f is a differentiable function, then $\frac{d}{dx}(x \cdot f(x^2)) =$

   (a) $f(x^2) + 2x^2 f'(x^2)$
   (b) $f'(2x)$
   (c) $f'(x^2) \cdot 2x$
   (d) $f'(3x^2)$
   (e) $f(x^2) + x \cdot f'(x^2)$

3. If $f(x) = \cos(-x)$, then the third derivative $f'''(x)$, is equal to:

   (a) $\sin(-x)$
   (b) $\cos(-x)$
   (c) $-\cos(-x)$
   (d) $-\sin(-x)$
   (e) $-\sin(x)$

4. Let $f(x) = \tan^3(x) + \tan\left(x^3\right)$. Find $\frac{df}{dx}$.

5. Suppose that $y = f(u)$ and $u = g(x)$ are differentiable functions of the input variables u and x respectively, and the image of g is contained in the domain of f. The derivative of the composite function $y = [f \circ g](x)$ at the input value $x = 2$ is given by the formula:

   (a) $[f \circ g]'(2) = f'(2)g'(2)$
   (b) $[f \circ g]'(2) = f'(g'(2))$
   (c) $[f \circ g]'(2) = f'(2)g(2) + g'(2)f(2)$
   (d) $[f \circ g]'(2) = f'(g(2))g'(2)$
   (e) $[f \circ g]'(2) = f(g(2))$